A GEOMETRICAL REPRESENTATION OF THE RELATIVE INTENSITY OF THE CONFLICT BETWEEN ORGANISMS.

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For our present purpose an organism may be thought of as a geometrical point in space. If such a point or organism is surrounded upon all sides by a homogeneous medium, such as air or water, it may be thought of as similarly related to the six faces of an enveloping or circumscribed cube, provided, the point or organism be placed at the point where the four diagonals bisect each other that pass through the cube from one to the other of its four pairs of trihedral angles or corners. The point or organism may be thought of as if placed at *o* in the diagram (fig. 1); the diagonals, which determine its position in such an ideal or imaginary cubic portion of space, will be, *ab*, *cd*, *ef* and *gh*. The relations of the point *o* would be equally well determined by the three axes, *x*, *y*, *z*, joining the centres each to each of the three pairs of faces of any such ideal enveloping cube. We may suppose further that the point or organism at *o*, if it moves from place to place, simply alters the position in space of the ideal enveloping cube of which it is always conceived to be the central point.

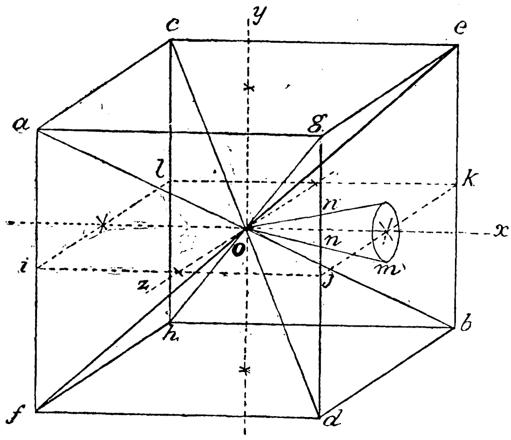

FIG. 1.

The possible number of ways of approach from every point on the surface of such a cubical fragment of space to the point

o at its centre will be $6a^2$, if it is assumed that there are a number of points on any and every edge, such as ac, of every one of the six faces of the cube. Since the enveloping cube has six faces, its cubic contents are equal to six square pyramids with the four sides of their bases with a length of a units or points, and with their vertices at o and with their bases formed of the six faces of the cube. If a represents the number of points lying in one of the edges of a side of the cube, it is obvious that the possible number of paths of approach toward o from all points at the surface of the enveloping cube must be $6a^2$, that is the whole number of points found in the bases of the six pyramids forming the sides of the enveloping cube.

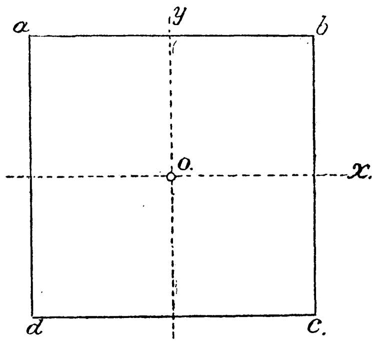

FIG. 2.

If we now suppose a fish swimming in water or a gnat flying in the air, in the same relations to a cubical fragment of its surroundings, as represented in the diagram, (fig. 1) or relatively to the faces of a cubical envelope of water or air, as o is to the six faces of the cube, *acge*, *agfd*, *gedb*, *fhdb*, *acfh* and *cehb*, we can, by assigning some definite numerical *value* to a, the length of any and every edge of the enveloping cube, as ac, for example, determine the number of directions in which it can be assailed by its enemies from the outside of its cubical envelope of air or water. If $a=100$, $6a^2=60{,}000$, so that any form swimming in water or flying in air is liable to be approached by enemies under such conditions as will amount to 60,000 possibilities of attack.

As a second supposable case, and if the point o were placed on a horizontal and impenetrable plane, cutting the enveloping cube into two equal parts through its four vertical sides along the lines *ijkl*, such a plane coinciding furthermore with the two horizontal axes x, z, of the cube, then would the point or organism o be accessible only from any direction lying in the upper face *acge* of the enveloping cube or the upper halves of

its four vertical sides *agij*, *acil*, *celk*, *gejk*. Then also, would the point or organism *o* be accessible only from $a^2+4\,(\frac{a}{2}a)=3a^2$ points or from only half as many as in the preceding case. That $3a^2$ must represent the possible number of paths along which *o* may now be approached, must be self-evident from the fact that the plane through x and z divides the ideal enveloping cube supposed in the first case into similar and equal halves, since one-half of $6a^2=3a^2$. If an organism is supposed to lie or move at *o* on the plane determined by *ijkl*, on the ground for instance, as a reptile, or at the bottom of the sea as a flounder, then will the possibilities of attack by enemies, with the factor a still equal to 100, be only 30,000.

A third case may be supposed where the point *o* is placed in the centre of a square plane with four equal sides (fig. 2) *ab*, *bc*, *cd* and *da* and with axes x and y across its two dimensions. Here if the number of points in any side of the square are a as before the number of points of approach will obviously be 4a, since there are as many pencils of lines converging at *o* as there are sides, namely, four. If, as in the case of heavy terrestrial organisms, attack by equally heavy or formidable enemies is only possible from every direction on a plane and not from every point on the surface of the whole or upper half of an enveloping cube, the possibilities of attack now sink to 400 or to only $\frac{1}{150}$th of the number in the first supposed condition and $\frac{1}{75}$th in the second.

A fourth case may be supposed where the point *o* may lie in the centre of a plane surface, which is perforated at the same point more or less deeply, so that *o* may, if it be a sensitive organism, retreat more or less into such perforation or cavity, now supposed to be excavated in a solid substratum. The small opening, as indicated in Fig. 2, into which *o* may retreat obviously represents only a very small part of the plane *abcd* and *o* is now accessible to an enemy only through some fraction of the number of points represented by a^2. This is still better shown in Fig. 1 where *m* is the circular periphery of the opening in one of the faces of the now solid cube enveloping *o*, on all sides except one, *o* now lying at the bottom of a cavity with parallel or converging sides *nn*. The accessi-

bility of o now becomes much reduced or only through m. If the lines nn Fig. 1 are produced we would have as the measure of accessibility of o, if, say the diameter d of m were $\frac{1}{5}$ of a, the number of points in jk, the number of points in d would be 20, the square of $\frac{1}{2}$ of 20, or the square of the radius of the circle m into π gives us the number of points in the area of m, which is 314 reckoning upon the basis of the arbitrary value assigned to a from the beginning. If, furthermore, still reckoning upon the same basis, we were to suppose the diameter of m to embrace only two points, then $r^2=1$ and the number of points of approach toward o would be only 3+. In this way by diminishing the diameter of m zero would be rapidly approximated and the accessibility of the organism at o become more and more difficult and greater and greater protection ensue against the attacks of enemies.

A fifth case may be supposed in which a cover may be developed or manufactured by the organism to close up the opening m supposed to exist in the preceding case, such as the lid made by a trap-door spider to close the entrance to its burrow. Other similar cases are presented by the test-bearing, univalve, operculate mollusks, the tubicolous and operculate worms and protozoa, or the valves of lamellibranchs or cirrhipeds. In such cases an approach is made toward total inaccessibility, the number of paths of approach and consequently of attack practically vanish to zero for all attacking forms which cannot bore into or crush the shelly covering of such prey.

The application of such geometrical conceptions and algebraic formulæ to represent the relative intensity of the struggle of organisms amongst themselves, under diverse relations to space, surfaces and cavities, must be obvious, if the point o be regarded as an organism and accessible to attack from $6a^2$ or 60,000 possible directions in the first case, from $3a^2$ or 30,000 possible directions in the second, from $4a$ or 400 in the third, from 314 to 3 in the fourth and from 0 in the fifth.

A condition similar to the fifth obtains where mimetic coloration exists. We may conceive an organism at the point o on a plane, such as a mimetically colored flounder assuming the tints of the plane upon which it rests, or a mimetic butterfly

or other organism at some point *o* in space, where the surroundings render mimetic coloration useful and where such surroundings now have tints so like the organism itself as to amount to positive and absolute concealment so long as the organism is quiescent. In this case accessibility to enemies again sinks to zero.

Parasites also are protected not only by virtue of their concealment within their hosts but also by the possible mimetic coloration of the latter.

Recapitulating, our series gives us the following comparative values:

For organisms swimming in water or flying in air, we may say that their accessibility *inter se* and liability to attack is from 60,000 directions.

For sessile organisms or those lying on a plane, their accessibility *inter se* and liability to attack is from 30,000 directions.

For heavy terrestrial forms their accessibility *inter se* and liability to attack by their fellows is from 400 directions.

For burrowing or tubicolous forms accessibility sinks to any where from 314 to 0 directions of approach.

For testaceous, operculate, mimetic or parasitic forms accessibility to enemies sinks to almost or quite 0.

That the intensity of the struggle for existence under the diverse conditions supposed varies as greatly as is indicated by the figures seems to be in a great measure supported by the facts of adaptations. For example, the high specialization of flying and free-swimming organisms must at once appeal to us in verification of the preceding statement. The high temperature of birds, the pneumaticity and specialization of their skeletons; the somatic, tracheal respiration and relatively high temperature of the bodies of flying arthropoda is proof of the high rate at which energy is dissipated and the effectiveness of the mechanism through which such dissipation is effected. Similarly, it may be said that only the free swimming types of fishes, such as the herrings, mackerels, sharks, etc., are "clipper built" for high velocities of motion while those that depend upon stealth, concealment for the capture of their prey and escape from enemies, are either depressed in form or even

much flattened, as flounders, for example, besides being usually mimetically colored. In these cases the figure of the body is so obviously correlated respectively with a capacity for a high velocity of motion and a capacity for only a low velocity of motion that there seems to be reason to suspect that the figure of the body is also correlated with a widely varying intensity of conflict with enemies and conditions in the struggle for existence, such as seems to be established by the various geometrical laws of their space relations during that conflict or struggle. If there have been forms which have developed in such directions as to give them greater celerity and consequently greater command over their surroundings in every direction there have been others which have been equally successful, often by the aid of mimicry, in getting into out-of-the-way corners and hiding-places in Nature where the possible number of approaches from their enemies have been also greatly reduced. Which of the two is the most advantageously situated it would be difficult to decide. For, while the swift and alert type must expend a great amount of energy in motion, the sluggish and concealed must vegetate and in a sense continually tend to degenerate in some one or other respect. The comparative rarity with which free-swimming or pelagic forms develop a tendency to bud or throw out stolons is perhaps to be connected with the great amount of energy expended in setting up motion. Where such colonial forms are mobile, most, be it observed, are obviously adapted as colonies for such motion, as the *Siphonophora*, chain *Salpæ* and *Pyrosoma*, for example. In other cases: sessile *Protozoa Porifera*, *Cœlenterata*, *Tunicata*, loss of active, free motility seems to end in a tendency to produce buds and stolons and develop colonial forms. It would seem as if the material and energy expended by the freely moving forms in active motion prevented the development of stolons and coherent colonies, intensified their specialization for active motion, and in some cases reduced their fertility. In the case of sessile forms or those with quiescent habits it would seem that the consequent saving of the material and energy of motion was compensated

by the development of colonies, buds, stolons or increased fertility.

The numerical series representing the gradual diminution of the intensity of the struggle of animal organisms amongst themselves, passing from very active, free moving forms to sessile and concealed forms is: 60,000, 30,000, 400, 314–3, 0. These marked contrasts seem to be well founded and highly significant. They probably indicate that in a completed theory of organic evolution, the rate at which energy is dissipated in the form of motion by a given animal organism must by taken into account. The possible number of directions of motion and attack under different conditions, it is scarcely necessary to add, have here been calculated upon the basis afforded by modern geometry, from certain relations of the point and line.